Market simulation with hierarchical information flux

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Abstract: We assume the market price to diffuse in a hierarchical comb of barriers, the heights of which represent the importance of new information entering the market. We find fat tails with the desired exponent for the price change distribution, and effective multifractality for intermediate times.

Keywords: Econophysics, Monte Carlo simulation, hierarchies, fat tails, multifractality

Among the widely believed stylized facts of real stock markets are fat tails, corresponding to a cumulative distribution function of price changes r decaying as $1/r^3$, and multifractality, stating that the exponent of the q-th moment of this distribution is not a linear function of q. We present here a simple model reproducing both properties.

We start with a hierarchical "comb" of barriers $b(x)$ symbolizing information relevant for the market, as a function of the current price x arbitrarily normalized to $0 < x < 1$. If this unit interval is divided into 2^n small intervals (we took $n = 20$), then at $x = 1/2$ the barrier has height $b(x) = n$; at $x = 1/4$ and $x = 3/4$ is has $b = n - 1$; at $x =$ multiples of $1/8$ not used before it has $b = n - 2$; at $x =$ multiples of $1/16$ not assigned before we set $b = n - 3$, and so on until $b = 1$. The remaining x values have $b = 0$.

Now the price x, starting in the middle, makes an unbiased random walk, hindered by the barrier $b(x)$. (More precisely, x is proportional to the logarithm of the price. Introducing a bias did not change out fat tails.) It overcomes this barrier with a probability $\exp(-b/2.2)$ and jumps, in case it overcomes the barrier, by the amount $2^{1-n}\exp(b/2.2)$. Thus large b correspond to very rare and very important informations, shifting the market appreciably, randomly up or

down. (To reduce artificial discontinuities, the actual barrier heights b where not set to integer values $b = k$ but taken as random between $k - 1$ and k.)

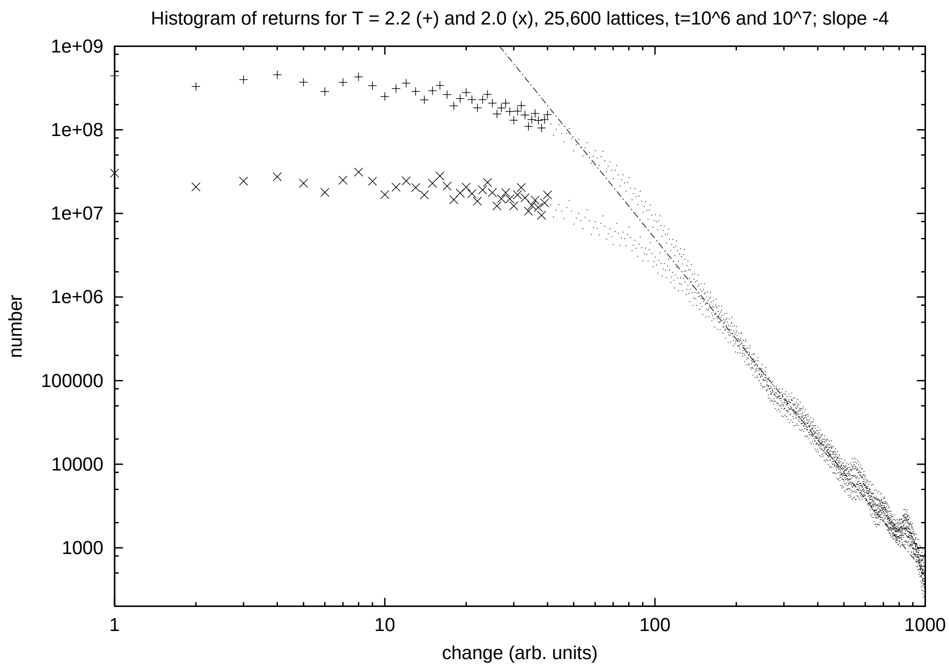

Figure 1: Fat tails in the histogram of price changes.

Each time step (”day”) corresponds to 100 random microsteps (”trades”) and results in one value $x(t)$ as a function of time t. The returns $r = x(t + 1) - x(t)$ are accumulated over $10^6 \dots 10^7$ time steps, and about $10^3 \dots 10^4$ independent samples. Figure 1 shows that the probability distribution function of these returns follows roughly for large r a power law with exponent -4, in agreement with reality [1, 2]. (If we change the ”temperature” parameter 2.2 also this exponent may change.) By definition, positive and negative changes appear equally often and equally strong.

For multifractality [3], we generalize the returns to $r_\tau = x(t + \tau) - x(t)$ and look at the moments

$$< r_\tau^q > \propto \tau^z$$

with exponents $z(q)$ describing the variation of the returns r_τ as a function of the time difference τ. Fitting these exponents in the time interval $100 < \tau < 1000$, Fig.2 shows $z(q)$ first to increase

with q and then to remain roughly constant ("multifractal"), while using less data for longer times, $1000 < \tau < 10000$, gives a nearly linear increase of z with q ("monofractal").

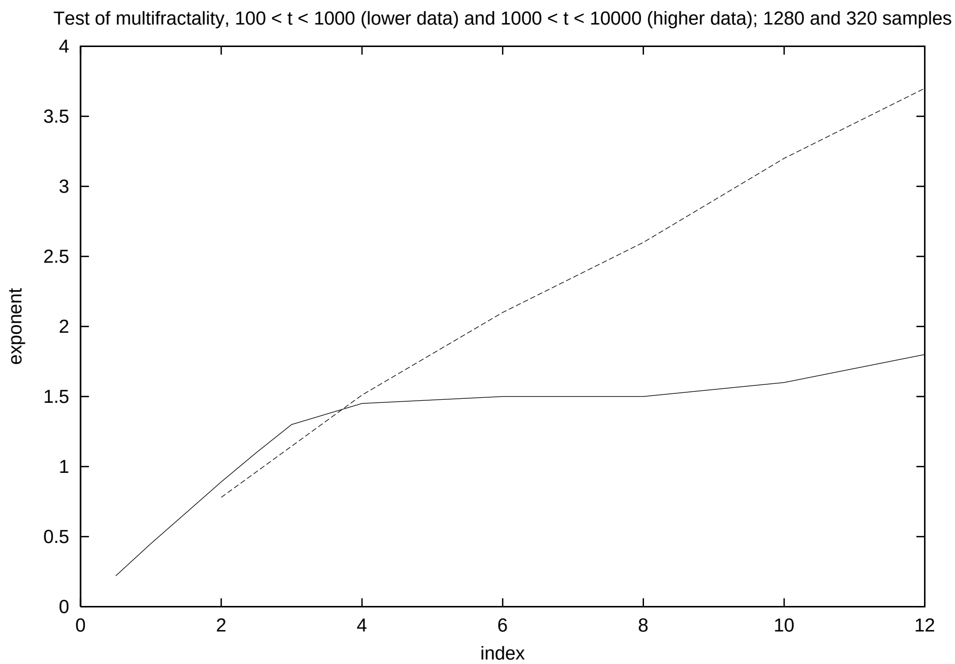

Figure 2: Exponent z versus index q for intermediate and for long times.

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